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Determining the objective function for geophysical joint inversion

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A serious problem in joint inversion is that of selecting those particular weight factors which control the correct contribution of each individual data set to the joint objective function. These weight factors are necessary input parameters of joint inversion but generally they are unknown. A method is proposed in which there is no need to know these weight factors. The maximum likelihood principle is applied so that optimization is done not only by the model parameters but also by the standard deviations of the data. The application is demonstrated by joint inversion of various simulated erroneous data sets. Linear toy examples of two data sets were studied for various common and uncommon model parameters. Experience has shown that with increasing number of data the estimates of the model parameters and the standard deviations of the data approximate their true values. Several simulated geophysical joint inversion problems were studied: gravity and magnetic, log evaluation, and refraction seismic. These examples show the applicability of the method.

Keywords: joint inversion, gravity, magnetics, log evaluation, refraction seismics

1. Introduction

Inversion of various data sets can be done independently (sequentially) for each type of measurement or it can be done jointly. Joint inversion is widely used because it produces mutually consistent estimates of the various unknown parameters [JULIÀ et al. 2000]. The purpose of joint inversion is that one objective function to be optimized is produced from the individual objective functions representing the various data sets. The usual strategy is to add the objective functions that create the joint objective function, but the magnitudes and the dimensions of the component objective functions are different. To overcome this difficulty, the difference between the observed data and the computed theoretical value can be normalized by the measured one. DOBRÓKA et al. [1991] and DE

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NARDIS et al. [2005] utilized this strategy. A further problem is that the quality of the different types of measurement may differ, therefore the component objective functions should be multiplied by weight factors thereby giving them the correct contribution for determining the model parameters. JULIÀ et al. [2000] introduced the 'influence parameter' p, which is responsible for the correct contribution between the seismic receiver function and the dispersion data sets. A similar method was applied in MOTA, MONTEIRO DOS SANTOS [2006], where resistivity and seismic velocity data were inverted jointly with weights α and $1-\alpha$ for the resistivity data and velocity data respectively. The parameters p and α are determined by repeated inversion experiments. There is no general rule for selecting these weight factors even though the results depend on them. TREITEL and LINES [1999] mentioned it as a big unsolved problem of joint inversion. A method is proposed here in which there are no input weight factors and the component functions provide the correct contribution to the joint objective function.

2. Method

Let us consider two different types of geophysical measurement. The sets of data and the corresponding sets of theoretical functions relating to the model are denoted by the vectors $\mathbf{d}_1$, $\mathbf{G}_1(\mathbf{m})$ and $\mathbf{d}_2$, $\mathbf{G}_2(\mathbf{m})$ respectively. They are referenced herein as task 1 and task 2. The dimensions of the vectors are n_1 and n_2. The unknown model parameters to be determined are the components of the vector $\mathbf{m}$, the dimension of which is n_m. The following scheme describes the connection between the measured and the theoretical quantities for task 1 and task 2:`

$$\mathbf{d_1} \quad \mathbf{G_1(m)} \quad \mathbf{e_1} \tag{1}$$

and

$$\mathbf{d_2} \quad \mathbf{G_2(m)} \quad \mathbf{e_2} \quad . \tag{2}$$

In the above equations, vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ represent the noise, these vectors are independent random numbers. It is assumed that they show normal distribution with zero expected values. The standard deviation for the $\mathbf{e}_1$'s is $\ _1$ and for the $\mathbf{e}_2$'s it is $\ _{2.}$ It is important to note that these two standard deviations are also unknown. The usual form of the joint objective function of the L_2 norm is

$$\lambda = w_1 \sum_{i=1}^{n_1} (d_{1,i} - G_{1,i}(\mathbf{m}))^2 + w_2 \sum_{j=1}^{n_2} (d_{2,j} - G_{2,j}(\mathbf{m}))^2, \tag{3}$$

where w_1 and w_2 are the weight factors. The minimum of λ determines the model parameter estimates which depend also on the weight factors, but these weight factors are generally unknown. If the maximum likelihood principle [KENDALL, STUART 1967] is applied separately to task 1 and task 2, the likelihood functions l_1 and l_2 should be maximized with respect to **m**. The mathematical form of the likelihood functions is:

$$l_1 = f_1(d_{1,1},\mathbf{m}) f_1(d_{1,2},\mathbf{m}) \ldots f_1(d_{1,n_1},\mathbf{m}) \tag{4}$$

where

$$f_1(d_{1,i},\mathbf{m}) = \frac{1}{\sqrt{2\pi}\,\sigma_1} \exp\left(-\frac{1}{2\sigma_1^2}(d_{1,i} - G_{1,i}(\mathbf{m}))^2\right), \tag{5}$$

[MENKE 1989] and similarly:

$$l_2 = f_2(d_{2,1},\mathbf{m}) f_2(d_{2,2},\mathbf{m}) \ldots f_2(d_{2,n_2},\mathbf{m}) \tag{6}$$

where

$$f_2(d_{2,j},\mathbf{m}) = \frac{1}{\sqrt{2\pi}\,\sigma_2} \exp\left(-\frac{1}{2\sigma_2^2}(d_{2,j} - G_{2,j}(\mathbf{m}))^2\right). \tag{7}$$

By maximizing l_1 and l_2, two independent estimates of $\mathbf{m}_1$ and $\mathbf{m}_2$ are determined respectively. The aim of joint inversion is that one common estimate of **m** is to be determined which corresponds to both $\mathbf{d}_1$ and $\mathbf{d}_2$ data. Therefore the product $l = l_1 l_2$ should be maximized. The complete form of l is:

$$l = \frac{1}{(\sqrt{2\pi})^{n_1+n_2}\sigma_1^{n_1}\sigma_2^{n_2}} \exp\left(-\frac{1}{2\sigma_1^2}\sum_{i=1}^{n_1}(d_{1,i} - G_{1,i}(\mathbf{m}))^2 - \frac{1}{2\sigma_2^2}\sum_{j=1}^{n_2}(d_{2,j} - G_{2,j}(\mathbf{m}))^2\right) \tag{8}$$

Function l depends on the measured data, the model parameters, and the standard deviations. This is the joint objective function for task 1 and task 2. By maximizing l, the estimates of the model parameters are determined. The usual way to solve the problem is to minimize the negative logarithm λ of function l:

$$\lambda = -\ln(l) \tag{9}$$

that is

$$\lambda = \frac{n_1+n_2}{2}\ln(2\pi) + n_1\ln\sigma_1 + n_2\ln\sigma_2 + \frac{1}{2\sigma_1^2}\sum_{i=1}^{n_1}(d_{1,i}-G_{1,i}(\mathbf{m}))^2 + \frac{1}{2\sigma_2^2}\sum_{j=1}^{n_2}(d_{2,j}-G_{2,j}(\mathbf{m}))^2 . \tag{10}$$

If the standard deviations σ_1 and σ_2 are known, the minimization relates only to the rightmost two members of eq. 10, which is equivalent to the minimization of λ in eq. 3. On comparing eqs. 3 and 10 it can be seen that the true values of the weights are:

$$w_1 = \frac{1}{2\sigma_1^2} \quad \text{and} \quad w_2 = \frac{1}{2\sigma_2^2} . \tag{11}$$

If the standard deviations are unknown, the minimization must also be done with respect to σ_1 and σ_2. The necessary conditions are:

$$\frac{\partial\lambda}{\partial\sigma_1} = 0 \quad \text{and} \quad \frac{\partial\lambda}{\partial\sigma_2} = 0 . \tag{12}$$

If these conditions are fulfilled, one gets:

$$\sigma_1^2 = \frac{1}{n_1}\sum_{i=1}^{n_1}(d_{1,i}-G_{1,i}(\mathbf{m}))^2 \tag{13}$$

and

$$\sigma_2^2 = \frac{1}{n_2}\sum_{j=1}^{n_2}(d_{2,j}-G_{2,j}(\mathbf{m}))^2 . \tag{14}$$

On inserting $_1$ and $_2$ from eqs. 13 and 14 into the expression of λ in eq. 10 and neglecting its constant part, the function will have the form:

$$\frac{n_1}{2}\ln\frac{1}{n_1}\,{}_{i\ 1}^{n_1}(d_{1,i}\quad G_{1,i}(\mathbf{m}))^2\qquad\frac{n_2}{2}\ln\frac{1}{n_2}\,{}_{j\ 1}^{n_2}(d_{2,j}\quad G_{2,j}(\mathbf{m}))^2\ . \quad (15)$$

This function does not include $_1$ and $_2$. The next step is to find the minimum of λ, this being a function of the model parameters $\mathbf{m}$. Once this minimum is found, it is the minimum with respect to $_1$ and $_2$ too, because of the constraints of eq. 12. Then the estimates of $_1$ and $_2$ can be calculated from eqs. 13 and 14. Their only further role is in the calculation of the standard deviations (m_i), which measure the uncertainty of the model parameter estimates. As can be seen, there may be more than two tasks for which the evaluation in this way can be done jointly.

3. Numerical examples

3.1. Toy example

Numerical calculations were done to determine whether the method reproduces $_1$, $_2$ and $\mathbf{m}$ for a known model with reasonable accuracy. For task 1 and task 2 linear direct problems $\mathbf{G}_1(\mathbf{m})$ and $\mathbf{G}_2(\mathbf{m})$ — without any geophysical meaning — were created and n_m=5 model parameters were chosen whose exact numerical values are: m_1=1, m_2=2, m_3=3, m_4=4 and m_5=5. The $\mathbf{d}_1$ and $\mathbf{d}_2$ data were produced on the basis of eq. 1, where $\mathbf{e}_1$ and $\mathbf{e}_2$ are random samples from two univariate normal distributions of zero expected values with $_1$=1 and $_2$=10. In *Fig. 1* the curves show that the calculated standard deviations approximate the exact values of $_1$, $_2$ with increasing values of n_1 and n_2.

In the next example $_1$=1 and $_2$=10, and then $_1$=1 and $_2$=100 standard deviations were chosen respectively. The number of data are n_1=35, n_2=50. Because different tasks may depend on different parameters of the model, three cases, viz.),) and) were studied. In case) task 1 and task 2 depend on all five model parameters; in case), task 1 depends on m_1, m_2, m_3, m_4; task 2 depends on m_2, m_3, m_4, m_5. In case) task 1 depends on m_1, m_2, m_3; task 2 depends on m_3, m_4, m_5. It means that in cases

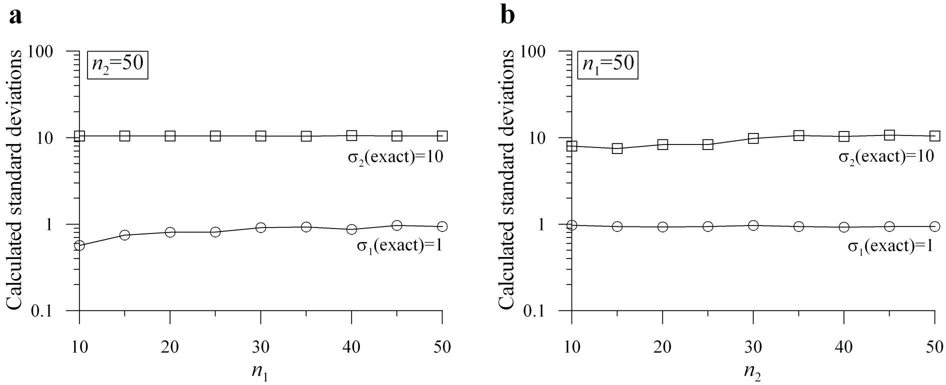

Fig. 1. Calculated standard deviations are plotted against the numbers n_1 and n_2. The exact standard deviations are $_1$(exact)=1 and $_2$(exact)=10. On graph a) n_2=50 while n_1 is increasing from 10 to 50, on graph b) n_1=50 and n_2 is increasing from 10 to 50

1. ábra. Számított standard szórás értékek függése az adatok n_1 és n_2 számától. A szórások egzakt értékei: $_1$(exact)=1 és $_2$(exact)=10. Az a) ábrán n_2=50, míg n_1 értéke 10 és 50 között változik. A b) ábrán n_1=50 és n_2 értéke változik 10 és 50 között

) and) there are respectively three and two common parameters, and for) there is one.

The uncertainty of the parameter estimates is measured by their standard deviations (m_i) and by comparing the exact and the estimated values. The results are given in *Table I.* In each part of the table there are two inversion results: one for $_1$=1 and $_2$=10 and the other for $_1$=1 and $_2$=100. In cases) all the model parameter estimates have almost the same standard deviations in spite of the fact that in the second example $_2$ is ten times greater than in the first example. In cases) the estimates of m_1, m_2, m_3, and m_4 show almost the same standard deviations as in cases), the standard deviation of m_5 is greater. The increase of $_2$ does not affect the standard deviations of m_1, m_2, m_3 and m_4, only the estimate of m_5 becomes even less accurate. In cases) the standard deviations of the estimates of m_1, m_2 and m_3 are approximately equal with those in cases) and). The standard deviations of m_4 and m_5 are greater and they are even greater for greater $_2$. Summarizing the results, one may conclude that the increase of $_2$ does not change the quality of the estimates of those model parameters which are variables of task 1, regardless of the fact that they may also be found in task 2. The increase of $_2$ causes a quality decrease for the estimates of parameters which are variables of task 2 exclusively.

a)

case α)	σ_1(exact)=1.0			σ_2(exact)=10.0	
n_1=35, n_2=50	σ_1(est)=0.960			σ_2(est)=10.46	
m(exact)	1.	2.	3.	4.	5.
m(est)	0.953	2.032	3.021	4.040	4.964
σ(**m**)	0.042	0.045	0.051	0.053	0.046
n_1=35, n_2=50	σ_1(exact)=1.0			σ_2(exact)=100.0	
case α)	σ_1(est)=0.959			σ_2(est)=104.8	
m(exact)	1.	2.	3.	4.	5.
m(est)	0.951	2.034	3.020	4.036	4.969
σ(**m**)	0.043	0.045	0.051	0.053	0.046

b)

case β)	σ_1(exact)=1.0			σ_2(exact)=10.0	
n_1=35, n_2=50	σ_1(est)=0.966			σ_2(est)=10.32	
m(exact)	1.	2.	3.	4.	5.
m(est)	0.939	2.030	3.011	4.035	4.770
σ(**m**)	0.040	0.045	0.049	0.051	0.147
n_1=35,n_2=50	σ_1(exact)=1.0			σ_2(exact)=100.0	
case β)	σ_1(est)=0.965			σ_2(est)=103.4	
m(exact)	1.	2.	3.	4.	5.
m(est)	0.941	2.033	3.011	4.030	3.288
σ(**m**)	0.040	0.045	0.049	0.051	1.424

c)

case γ)	σ_1(exact)=1.0			σ_2(exact)=10.0	
n_1=35, n_2=50	σ_1(est)=0.969			σ_2(est)=10.18	
m(exact)	1.	2.	3.	4.	5.
m(est)	0.947	2.038	3.023	4.451	4.415
σ(**m**)	0.038	0.044	0.043	0.364	0.352
n_1=35, n_2=50	σ_1(exact)=1.0			σ_2(exact)=100.0	
case γ)	σ_1(est)=0.969			σ_2(est)=101.8	
m(exact)	1.	2.	3.	4.	5.
m(est)	0.947	2.037	3.024	8.622	-0.773
σ(**m**)	0.038	0.044	0.044	3.633	3.516

Table I. a-c. Joint inversion results of two data sets for cases α), β) and γ). The table contains the numbers n_1 and n_2 of the data sets, the exact and estimated values of $_1$ and $_2$, the exact and estimated values of the model parameters **m**, and their standard deviations

I. a–c. táblázat. Két adatrendszer egyesített inverziós eredményei láthatók az α), β) és γ) esetekre. A táblázat tartalmazza az adatok n_1 és n_2 számát, a $_1$ és $_2$ standard szórások egzakt és becsült értékeit valamint az **m** modellparaméterek egzakt és becsült értékeit és azok standard szórásait

3.2. Joint inversion of gravity and magnetic data:

The gravity problem is task 1 and the magnetic problem is task 2. The geological object is a spherical body with homogeneous density and homogeneous intensity of magnetization. The radius of the body is $r = 50$ m. The mass of the sphere is $m = 2\text{x}10^8$ kg (=2x10^5 t), so the density of the sphere is 381.97 kg/m^3. The vertically oriented magnetic moment of the sphere is M=5x10^6 Am^2, the intensity of magnetization is 9.549 A/m. The z axis of the rectangular coordinate system is directed downwards. The coordinates of the centre of the body are x_0=0, y_0=0 and z_0=100 m. Both the gravity and the magnetic data relate to the horizontal plane z=0. The g gravity anomaly is given in microgal units and the B_z magnetic flux density anomaly is given in nT units. The theoretical model for gravity data is

$$g \quad G\frac{m}{r^2}\frac{z_0}{r} \quad , \tag{16}$$

where

$$r \quad ((x \quad x_0)^2 \quad (y \quad y_0)^2 \quad z_0^2)^{1/2}$$
,

which is the distance between the centre of the sphere and the point of observation, and G=6.673 *10^{-11} m^{3}kg^{-1}s^{-2} is the constant of gravitation. For magnetic data [PARASNIS 1979] the theoretical model is:

$$B_z \quad \frac{\ \ _0}{4}\frac{M}{r^3} \quad 3\frac{z_0^2}{r^2} \quad 1 \quad . \tag{17}$$

In the above equation μ_0 is the magnetic permeability of the vacuum, $\mu_0 = 4\pi * 10^{-7}$ VsA^{-1}m^{-1}.

The measuring area is a square containing 21*21=441 points, the distance between the neighbouring points is 10 m in both the x and y directions. The spherical body is below the centre of the square. The maximum theoretical anomaly values are 133.46 microgal and 1000 nT. At each point both gravity and magnetic data are measured. The unknown parameters of the model are mass m, magnetic moment M, and the coordinates of the centre of the spherical body x_0, y_0 and z_0.

The erroneous $\mathbf{d}_1$ and $\mathbf{d}_2$ data were produced as in the earlier examples; the standard deviations for gravity and magnetic data are denoted by $_{grav}$ and $_{magn}$ respectively. Many inversion calculations were done for different values of $_{grav}$ and $_{magn}$. Two inversion results are shown in *Table II*. It is shown that the estimates of $_{grav}$ and $_{magn}$ approximate their exact values. The model parameter estimates and their standard deviations depend on the magnitude of $_{grav}$ and $_{magn}$.

a $_{grav}$(exact) = 5 μgal		$_{magn}$(exact) = 20 nT	
$_{grav}$(est) = 5.22 μgal		$_{magn}$(est) = 20.44 nT	
parameter	exact	estimated	
Δm (t)	200000	199770	960
M (Am^2)	5000000	4946600	30000
x_0(m)	0	–0.14	0.256
y_0(m)	0	0.07	0.256
z_0(m)	100	99.59	0.268

b $_{grav}$(exact) = 10 μgal		$_{magn}$(exact) = 5 nT	
$_{grav}$(est) = 10.53 μgal		$_{magn}$(est) = 5.06 nT	
parameter	exact	estimated	
Δm (t)	200000	201710	1400
M (Am^2)	5000000	4989100	7990
x_0(m)	0	0.02	0.08
y_0(m)	0	–0.01	0.08
z_0(m)	100	99.93	0.07

Table II. a-b. Joint inversion results of simulated erroneous gravity and magnetic measurements. In the examples there are the exact and estimated values of the standard deviations $_{grav}$ and $_{magn}$, the exact model parameters, their estimated values, and the standard deviations of the estimates.

II. a–b. táblázat. Példák hibával terhelt szimulált gravitációs és mágneses adatok egyesített inverziójára. A táblázatok tartalmazzák a $_{grav}$ és $_{magn}$ adatok egzakt és becsült értékeit, az egzakt és becsült modellparamétereket és a becsült paraméterek standard szórásait.

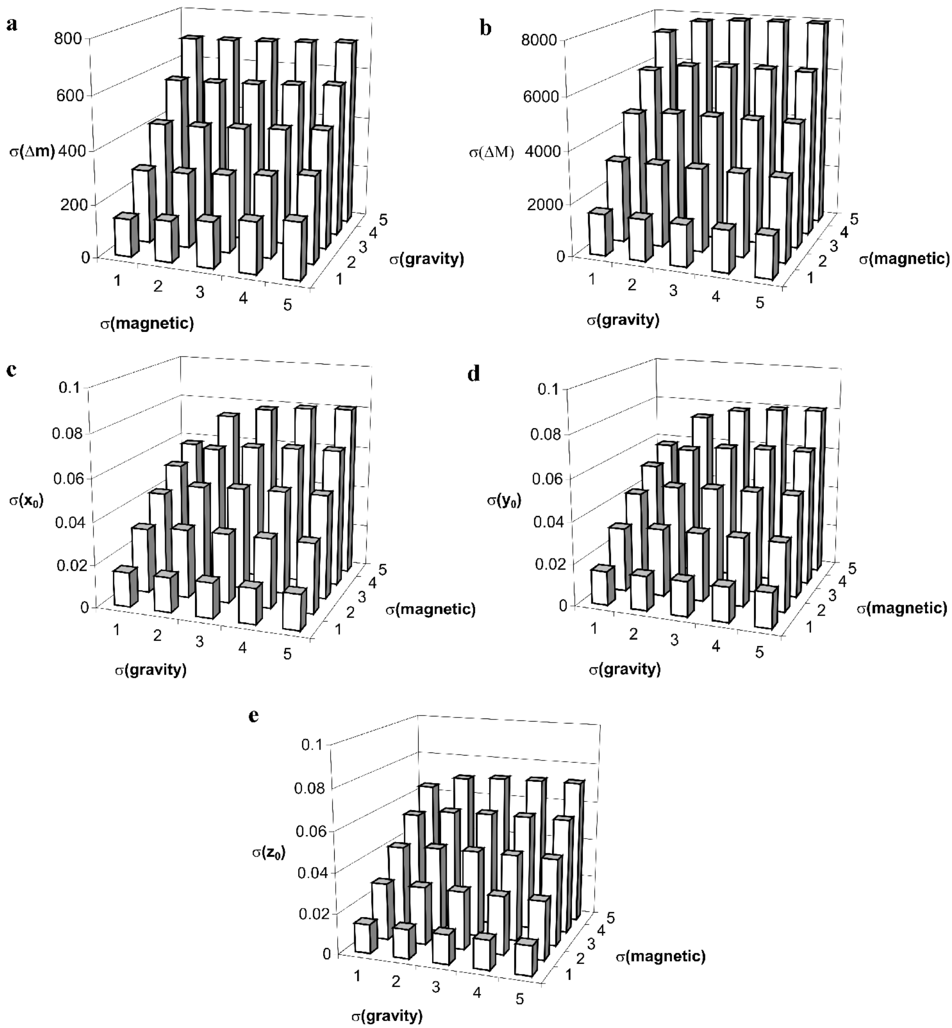

Fig. 2. Standard deviations $_{\Delta m}$, $_{\Delta M}$, $_{x_0}$, $_{y_0}$ and $_{z_0}$ are plotted on graphs a), b), c), d) and e) respectively for the values of $_{grav}$ = 1, 2, 3, 4 and 5 μgal and $_{magn}$ = 1, 2, 3, 4 and 5 nT. The graphs show the influence of $_{grav}$ and $_{magn}$ on the results of the joint inversion

2. ábra. Az a), b), c), d) és e) grafikonok a számított $_{\Delta m}$, $_{\Delta M}$, $_{x_0}$, $_{y_0}$ és $_{z_0}$ standard szórásoknak a $_{grav}$ és $_{magn}$ szórások nagyságától való függését mutatják $_{grav}$ = 1, 2, 3, 4 és 5 μgal és $_{magn}$ = 1, 2, 3, 4 és 5 nT értékek esetére

We should now like to deal with the problem how the standard deviations $_{grav}$ and $_{magn}$ influence the accuracy of the model parameter estimates. Systematic inversion experiments were done for $_{grav}$= 1, 2, 3, 4 and 5 microgal and $_{magn}$= 1, 2, 3, 4 and 5 nT. The standard deviations (m), (M), (x_0), (y_0) and (z_0) are illustrated on plots **a**, **b**, **c**, **d** and **e** respectively in *Fig. 2*. The standard deviation (m) shows essential dependence on $_{grav}$, the effect of $_{magn}$ is relatively negligible. Similarly, (M) depends essentially on $_{magn}$ and the effect of $_{grav}$ is negligible. The structure of plots **c**, **d**, **e** is similar to that of plot **b**, where stronger dependence on the geometrical parameters according to $_{magn}$ than $_{grav}$ can be seen. These examples show that in this special joint inversion problem the magnetic task produces a dominant influence on the estimates of the common parameters.

3.3. Joint inversion of penetration logs:

The evaluation of four penetration logs [FEJES, JÓSA 1990] is considered in this section: gamma ray (GR), density (DEN), neutron porosity (FIN), and resistivity (R). They are the four tasks of joint inversion. The model is a homogeneous soil layer consisting of water (V_w) and air (V_g) in the pore space, and a silica matrix (V_s) and clay (V_{cl}). There are n measuring depth points against the layer. The theoretical response functions of the logs [SERRA 1984] are:

$$GR \quad GR_s \quad V_{cl}\,(GR_{cl} \quad GR_s) \quad , \tag{18}$$

$$DEN \quad V_w \quad V_{cl}\,DEN_{cl} \quad (1 \quad V_w \quad V_g \quad V_{cl})\,DEN_s \quad , \tag{19}$$

$$FIN \quad V_w \quad V_{cl}\,FIN_{cl} \quad , \tag{20}$$

$$R \qquad \frac{V_{cl}}{R_{cl}} \quad \frac{V_w}{R_w}^{\;1} \quad \frac{1}{(V_w \quad V_g \quad V_{cl})^{me \;\; 1}} \quad . \tag{21}$$

Eq. 21 is deWitte's shaly sand resistivity model [DEWITTE 1950], where me represents the cementation exponent. The constant zone parameters in the theoretical response functions and their numerical values are:

GR_{cl}=120 API unit, gamma ray intensity of clay,
GR_s=50 API unit, gamma ray intensity of sand,
DEN_{cl}=2.1 g/cm^3, density of clay,
DEN_s=2.65 g/cm^3, density of sand,
FIN_{cl}=0.3, neutron porosity of clay,
R_w=10 ohmm, resistivity of water,
R_{cl}=5 ohmm, resistivity of clay.

The exact model parameters are:

V_w=0.10, V_g=0.15, V_s=0.50 and V_{cl}=0.25.

For the *GR*, *DEN*, and *FIN* data, normal distribution is assumed; for the *R* data lognormal distribution is assumed. The standard deviations for producing the simulated erroneous data are: (*GR*)=10 API units, (*DEN*)=0.05 g/cm^3, (*FIN*)=0.05 and (log*R*)=0.11. n=20 erroneous data were produced. The results of the evaluation are shown in *Table III*. Comparison of the results shows convincing similarity between the original and the estimated quantities.

	GR	*DEN*	*FIN*	log*R*
σ(exact)	10.0 API	0.050 g/ccm	0.050 decim.	0.110 decim.
σ(est)	9.33 API	0.051 g/ccm	0.049 decim.	0.138 decim.
model	V_w	V_g	V_s	V_{cl}
m(exact)	0.100	0.150	0.500	0.250
m(est)	0.113	0.140	0.496	0.251
(**m**)	0.014	0.008	0.010	0.016

Table III. Joint inversion log evaluation example of gamma ray (*GR*), density (*DEN*), neutron porosity (*FIN*) and resistivity (*R*) penetration logs relating to a homogeneous soil layer. The parameters of the model are: water content (V_w) and gas content (V_g) of the pore space, the amount of silica (V_s) and clay (V_{cl}). There are the exact and the estimated standard deviations of the logs, the exact and estimated values of the model parameters and their standard deviations

III. táblázat. Hibával terhelt szimulált mérnökgeofizikai szelvények homogén talajréteg modellre vonatkozó egyesített inverziója. A figyelembe vett szelvények: természetes gamma (*GR*), sűrűség (*DEN*), neutron-porozitás (*FIN*) és fajlagos ellenállás (*R*). A talajmodell paraméterei: talajvíz (V_w) és levegő (V_g) mennyisége a pórustérben, homok (V_s) és agyag (V_{cl}) mennyisége. A táblázat tartalmazza a szelvények egzakt és becsült standard szórásait, a paraméterek egzakt és becsült értékeit valamint azok standard szórás értékekeit

3.4. Joint inversion of two refraction seismic data sets

Both task 1 and task 2 are simulated refraction seismic inversion problems. Two data sets $\mathbf{d}_1$ and $\mathbf{d}_2$ were recorded at the same measurement site therefore the same earth model is used for them. It is assumed that they show different normal error distributions which are characterized by σ_1=1 ms (good quality data) and σ_2=5 ms (poor quality data) standard deviations respectively. The number of data are n_1=20 and n_2=60. The model consists of two layers which are separated by a horizontal plane. The theoretical t travel time formulae [LOWRIE 2007] are:

$$t = \frac{x}{v_1} \quad \text{if} \quad x < x_i \qquad \text{and} \tag{22}$$

$$t = \frac{x}{v_2} + 2h\frac{\sqrt{v_2^2 - v_1^2}}{v_1 v_2} \quad \text{if} \quad x \geq x_i \quad , \tag{23}$$

where x_i is the crossover distance:

$$x_i = 2h\sqrt{\frac{v_2 + v_1}{v_2 - v_1}} \quad . \tag{24}$$

The exact parameters of the model are: v_1=300 m/s, v_2=600 m/s and the depth of the layer boundary is h=5 m. The geophones are situated along a line so that for task 1 the distances measured from the shot point are x_1=1 m, 2 m, 3 m,…, 20 m and for task 2 x_2=2 m, 4 m, 6 m, …, 120 m respectively. The simulated erroneous data were produced as in the earlier examples. Four cases of inversion were done the results of which are in *Table IVa – d*:

case a): joint inversion for task 1 and task 2 according to equation 3 with w_1=w_2=1,

case b): joint inversion for task 1 and task 2 according to equation 15,

case c): separate inversion for task 1,

case d): separate inversion for task 2.

a) $n_1 = 20$ $_1$(exact) = 1 ms (est) = 3.62 ms		$n_2 = 60$ $_2$(exact) = 5 ms (est) = 3.62 ms	
parameter	exact	estimated	
v_1 (m/s)	300	293	8.8
v_2 (m/s)	600	590	7.2
h(m)	5	4.63	0.30

b) $n_1 = 20$ $_1$(exact) = 1 ms (est) = 0.99 ms		$n_2 = 60$ $_2$(exact) = 5 ms $_2$(est) = 5.65 ms	
parameter	exact	estimated	
v_1 (m/s)	300	300	2.1
v_2 (m/s)	600	594	5.5
h(m)	5	4.92	0.12

c) $n_1 = 20$ $_1$(exact) = 1 ms (est) = 0.99 ms		$n_2 = 0$	
parameter	exact	estimated	
v_1 (m/s)	300	300	2.0
v_2 (m/s)	600	417	68.0
h(m)	5	3.28	0.99

d) $n_1 = 0$		$n_2 = 60$ $_2$(exact) = 5 ms $_2$(est) = 5.60 ms	
parameter	exact	estimated	
v_1 (m/s)	300	287	19.5
v_2 (m/s)	600	590	8.7
h(m)	5	4.51	0.49

Table IV. a-d. The results of different inversion examples of two refraction seismic data sets are demonstrated. The tables contain the numbers of data n_1 and n_2, the exact and estimated standard deviations of the two data sets $_1$ and $_2$. There are the exact and the estimated values of the layer velocities v_1, v_2 respectively and those of the depth h of the boundary. The results are: usual joint inversion with equal weights (a), joint inversion according to equation 15 (b), and separate inversions of the two data sets (c) and (d)

IV. a–d. táblázat. A táblázatok két refrakciós adatrendszer inverziós eredményeit tartalmazzák. n_1 és n_2 a megfelelő adatszámok, $_1$ és $_2$ az adatrendszerek standard szórásai, v_1, v_2 a rétegek szeizmikus sebesség értékei, h a réteghatár mélysége. A táblázatokban a megfelelő mennyiségek egzakt és becsült értékei vannak négy inverziós megoldásra vonatkozóan: szokásos inverzió a (3) egyenlet szerint azonos súlyokkal (a), egyesített inverzió a (15) egyenlet szerint (b) és a két adatrendszer független inverziója (c) és (d)

In case a) the usual inversion strategy is used. Apparently there is no reason to apply weights because both data sets of task 1 and task 2 are seismic refraction travel times. In case b) the interpreter assumes that the quality of data may be different. Separate inversions of cases c) and d) were done to compare all the possible solutions. The best results (minimum standard deviations) were found for case b). Only the standard deviation of v_1 in case c) shows the same magnitude as in case b).

4. Conclusion

The proposed joint inversion method takes into consideration the 'forgotten part' of the likelihood functions. Optimization of the joint objective function relates not only to the unknown model parameters **m** but is extended to the unknown standard deviations. This is in accordance with the maximum likelihood principle, which asserts that the probability of observed data **d** is made as large as possible [MENKE 1989]. Optimization of the joint objective function according to the standard deviations of the data can be done analytically. This leads to a modified form of the joint objective function which depends only on the model parameters **m**. The method can be regarded as the generalization of the L_2 norm inversion technique. It is important to note that the data sets should be uncorrelated. The expression of the joint objective function of equation 15 can easily be utilized in the existing L_2 norm algorithm.

The simulated inversion examples dealt with relate to toy examples, gravity–magnetic joint inversion, log evaluation, and refraction seismic inversion. Based on experience the estimated standard deviations of the data approximate their exact values with a maximum of 10%–15% difference. To get this level of approximation the number of data in each task should be more than 10–15. The model parameter estimates approximate their known exact values with the degree measured by their calculated standard deviations. The example calculations show the influence of the data errors on the estimates of the common and uncommon model parameters of different tasks.

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Az egyesített geofizikai inverzió célfüggvényeinek meghatározása

DRAHOS Dezső

Az egyesített inverzió megvalósítása során probléma a különböző geofizikai mérések adatrendszereihez tartozó súlyfaktorok megválasztása. Ezek az inverzió szükséges bemeneti paraméterei, amelyek általában nem ismertek, de a megoldás ezeknek is függvénye. Jelen dolgozatban egy olyan megoldást mutat be a szerző, amely alkalmazása során nem szükséges ismerni a súlyfaktorokat. A módszer lényege, hogy a maximum likelihood becslés során nemcsak a modell-paraméterek szerint, hanem az adatok standard szórása szerint is optimalizálni kell a likelihood függvényt. A cikk geofizikai jelentés nélküli adatrendszerek, valamint szimulált, hibával terhelt geofizikai adatrendszerek együttes inverziós példáival illusztrálja a módszer alkalmazhatóságát.

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Dezső Drahos graduated as a geophysicist from the Eötvös Loránd University, Budapest in 1967. He received his Ph.D. degree from the Hungarian Academy of Sciences in 1989. From 1967 to 1970 he worked as a scientific associate at the Eötvös Loránd Geophysical Institute where he was involved in theoretical and experimental modelling of logging sondes. Since 1970 he has been at the Department of Geophysics of Eötvös Loránd University initially as a lecturer and, since 1991, as associate professor. Besides his teaching activities he is interested in geophysical inversion, well logging and petrophysics. He is a member of the Association of Hungarian Geophysicists, and also a member of EAGE, SEG and SPWLA.